PMM U.S.S.R.,Vol.49,No.2,pp.179-184,1985
Printed in Great Britain

0021-8928/85 $10.00+0.00
Pergamon Journals Ltd.

ON THE CONFIGURATION OF SHOCK WAVES ENCLOSING A LOCAL SUPERSONIC ZONE*

A.N. KRAIKO

The problem of shock waves enclosing a local supersonic zone (LSZ) is considered. The possibility of obtaining the "inverse λ-like" configuration is pointed out. The first shock in such a configuration appears inside the LSZ. An infinite discontinuity of the second derivatives propagates from the point of its origin along the c^+ characteristic and arrives at the sonic line (Sl). This in turn may lead to the appearance of a second shock forming, together with the first shock, the configuration in question.

1. Consider a flow in a LSZ (Fig.1) which appears in the case of transonic flows of a perfect (inviscid and non-heat conducting) gas past a profile or a solid of revolution (Fig. 1), and in the corresponding modes near the smallest cross-section of the Laval nozzle. In Fig.1 the gas moves from left and right, and the dashed, thick and thin curves depict, respectively, the sonic line, the shock wave and the characteristic.

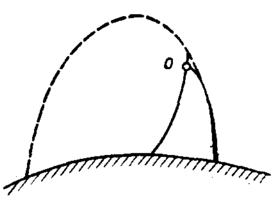

Fig.1

We know /1—7/ that in spite of the fact that, basically, a shock-free flow may emerge from a LSZ, continuous solutions are an exception and the LSZ is generally closed by one or more shock waves. Considerable effort was made in investigating the nature of such shocks to explain the possibility of so-called natural solutions in which no discontinuities of any derivatives whatsoever of the flow parameters arrive at the SL along the characteristics. An analysis carried out by Landau and Lifshitz /8/ and by Ludwig /9/ has shown the impossibility of the existence of a natural solution which would have a self-similar structure near the point at which the shock wave appears on the SL. Irrespective of this fact and by virtue of other factors (in particular, due to the insufficient incisiveness of the method of visualizing the weak shocks and the erroneousness of Theorem 8 of /1/ which states that the appearance of a shock within the LSZ is not possible), numerous attempts have been made /4, 10—18/ to construct natural solutions with a shock originating on the SL. All such attempts have failed.

On the other hand, using the assumption mentioned above concerning the self-similar character in the investigation of reflection of the singularities from the SL, brought in from the direction of the body along the c characteristic, it was found /8, 19—26/, that the closing shock can originate on SL only in the case when a sufficiently strong singularity arrives at it, such as a finite discontinuity of the first-order derivatives or an infinite discontinuity of the second-order derivatives of the flow parameters. As a result, the point of view of /9/ prevailed, according to which the shock closing the LSZ originates, provided that the body is sufficiently smooth, within it and is caused by the intersection of the characteristics (the second family in Fig.1) of the compression waves propagating from SL. This point of view is corroborated by numerical computations /27—30/. It is true that the authors of these papers occasionally state that the meshes used by them do not allow the fine flow structure to be resolved near the point ("tip") at which the shock originates (point o in Fig.1). The solutions of /31—33/ offer further evidence that a shock may originate within LSZ. The solutions constructed in hodograph variables show, on passing to the physical plane, a "fold" near the right-hand boundary of LSZ whose removal requires the introduction of a shock originating within the LSZ.

In the papers cited the singularities brought to SL invariably connected with the analogous singularities of the streamlined contour. It is for this reason that the only search which seemed to be justified, was that for the "natural" solution, in the case of the bodies which had no points, within the LSZ, of finite curvature discontinuity or of infinite discontinuity of its first derivative. The fact that the "discontinuous" characteristic may originate within the flow was however disregarded. In Fig.1 the c^+-characteristic passing through the point o will represent such a characteristic. If the discontinuity, which appears, is found to be of the required magnitude and sign, then another shock may "reflect" from the point of its intersection with the SL. Both shocks form an "overturned" or "inverse λ-like" configuration closing the LSZ.

*Prikl.Matem.Mekhan.,49,2,236-243,1985

On the weak shocks the discontinuity of the Riemann invariant corresponding to the characteristics of the "opposite" family (the first family in the case of Fig.1), as well as the discontinuity of the entropy s, are both proportional to the cube of the pressure increment $[p]$ (here and henceforth $[\psi] = \psi_+ - \psi_-$ is the difference in ψ to the right and left of the discontinuity). This at first glance implies (see e.g. /34/) that on the c^+-characteristic emerging from the point o the first and second derivatives of the parameters are continuous and no second shock can appear according to what was said above. This argument, however, would be valid if the point o were to lie on the discontinuous c^- characteristic carrying the jump in the curvature of the stream lines. Here $[p]$ is a linear function of the distance τ measured along the shock from its origin, with an accuracy up to higher-order infinitesimals. If on the other hand, as in the general case of a flow in LSZ, the shock is caused by the intersection of the "continuous" (along the curvature of the stream lines) c^--characteristics, then/35, 36/:$[p] \sim \tau^{1/2}$ and the conclusion arrived at in /34/ concerning the continuity of the second derivatives is found to be false. Moreover, in this situation an infinite discontinuity of the Riemann invariant I^+ corresponding to the c^+-characteristics, propagates from o along the c^+-characteristics.

2. When the characteristic carrying the infinite discontinuity in the second derivative of I^+ is reflected from SL where $p = \text{const}$, a shock wave will appear at the point of reflection provided that the sign of the discontinuity in question corresponds to the rarefaction wave. In this connection we must explain what type of discontinuity appears on the c^+-characteristic passing through the point o representing the tip of the inner shock. In explaining this we shall limit ourselves to plane, isoenergetic and isentropic (up to the shock) flows, although it can be shown that the basic result remains valid also in the general case of non-isoenergetic, non-isentropic and axisymmetric flow. We place the origin of Cartesian xy coordinates at the tip of the inner shock, and direct the x axis along the velocity vector $\mathbf{q}$. Let ϑ be the angle formed by $\mathbf{q}$ with the x axis, a the speed of sound, $M = q/a$ the Mach number, $(q = |\mathbf{q}|)$, $\alpha = \arcsin(1/M)$ the Mach angle and ρ the density. We shall, as a rule, denote the parameters at the point o by the subscript "o". The invariants I^+ and I^- which are preserved in the subregions of the continuity and isentropicity of the flow on the c^+ and c^- characteristics respectively, are given by the formulas

$$I^{\pm} = I^{\pm}(\vartheta, p) = \vartheta \pm \int_{p_0}^{p} \frac{\operatorname{ctg} \alpha}{\rho q^2}\, dp \tag{2.1}$$

with the natural correspondence of the indices and signs. The integral in (2.1) is computed for $s = s_0$ and is therefore a function of p only when p_0 is fixed. It can be shown that, irrespective of the presence of a curved shock, the invariants $I^{\pm}$ are also preserved in a small zone of weak non-isentropicity of the flow adjacent to the tip of the shock, with the accuracy required in future discussions. At the same time, the increments in I^+ and s must be taken into account during the passage through the shock.

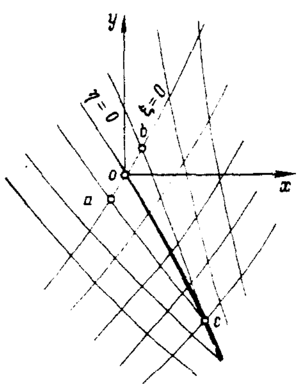

Fig.2

Fig.2 shows the mesh of the characteristics of both families and the shock near the origin of coordinates. As we know, one of the properties of weak shocks is the fact that they follow the path along the bisector of the angle between the characteristics of the same family arriving at it from the different sides. Therefore the tip of the shock divides the base ab of the characteristic curvilinear triangle in half (ab is a segment of the c^+ characteristic, ac and bc are the segments of the c^--characteristics). We introduce the characteristic variables $\xi\eta$ where $\xi = \text{const}$ along the c^+-characteristics. We choose them in such a manner that $\xi_0 = \eta_0 = 0$. We take the invariant I^- as η, with the invariant satisfying the conditions given and decreasing monotonically during the passage from a to b. The decrease is generally caused by the fact that the c^--characteristics generate a compression wave near o. In the case of a flow in LSZ, the monotonic decrease in I^- during the motion towards SL takes place on all c^+ characteristics /1, 2/. Since $\eta_o = I_o^- = 0$, it now follows that $\eta_a > 0$, and $\eta_b < 0$.

If we regard x and y as functions of ξ and η, then the following relations must hold at the tip of the shock arising within the perturbed flow /34/:

$$\frac{\partial x}{\partial \eta} = \frac{\partial y}{\partial \eta} = 0, \quad \frac{\partial^2 x}{\partial \eta^2} = \frac{\partial^2 y}{\partial \eta^2} = 0 \tag{2.2}$$

Therefore we have, on the characteristic $\xi = 0$ near the point o,

$$x = A\eta^3 - o(\eta^3), \qquad y = A \operatorname{tg} \alpha_0 \eta^3 - o(\eta^3), \quad A < 0 \tag{2.3}$$

where the constant A determines the compression of the flow leading to the appearance of the shock. Relation (2.3) and equality of the segments ao and ob imply that $\eta_a = -\eta_b$. Recalling that at the point c $[I^+] \sim [p]^3$, we find $[p] \sim [I^-] = [\eta] = 2\eta_b$. For the same reason the angle at the vertex c of the triangle abc is also of the order of η_b. Since we have, by virtue of (2.3) $l \sim \eta_b^3$ for the length l of the segment ab, this yields $\tau \equiv \tau_c \sim \eta_b^2 = [\eta]^2 / 4$. This together with the formula for $[p]$ given above, implies that $[p] \sim \tau^{1/2}$. Since the triangle abc is "normal" and its base is η_b-times smaller than its height $(|\eta_b| \ll 1)$, the basic result of the paper remains valid in the general case of non-isentropic, non-isoenergetic and axisymmetric flow. We note by the way that in case of the shocks generated on the discontinuous (with respect to the stream line curvature) c^--characteristics, only the first equalities in (2.2) hold. Therefore we have $l \sim [\eta]^2, \tau \sim [\eta]$ and unlike the situation under consideration, $[p] \sim \tau$ and $[I^+] \sim \tau^3$.

The sign of the singularity propagating along the characteristic $\xi = 0$ from the point o towards the SL, is governed by the sign of the coefficient G in the formula $[I^+] \simeq G\tau^{3/2}$ and $G = B^3C^3D$ provided that we write the relations obtained above in the form

$$[p] = B\,[\eta] + o\,([\eta]), \quad [\eta] = C\tau^{1/2} + o\,(\tau^{1/2}), \quad [I^+] = D\,[p]^3 + o\,([p]^3)\,. \tag{2.4}$$

We will find the coefficients of (2.3), but we will first show that the singularity on the line $\xi = 0$ is characterized when $\eta = \mathrm{const} < 0$ by zero discontinuity in the first and the second derivatives of I^+. To do this we shall take as ξ, τ of the point of intersection of the c^+-characteristics with the shock or with the c^- characteristic arriving at o. We shall take into account the fact that the variation in I^+ on the segment of the c^+-characteristic is of the order of $O\,(\tau^{3/2})$ in the region of non-isentropicity of the flow behind the shock. Then, by virtue of the fact that I^+ is conserved on the c^+ characteristics we have, for small $\xi \geqslant 0$,

$$\left(\frac{\partial I^+}{\partial \xi}\right)_+ = \left(\frac{\partial I^+}{\partial \xi}\right)_- + \frac{3}{2} G\xi^{1/2}, \quad \left(\frac{\partial^2 I^+}{\partial \xi^2}\right)_+ = \left(\frac{\partial^2 I^+}{\partial \xi^2}\right)_- + \frac{3}{4} G\xi^{-1/2}. \tag{2.5}$$

Here $(\partial I^+ / \partial \xi)_-$ and $(\partial^2 I^+ / \partial \xi^2)_-$ are regular functions of ξ which, as $\xi \to -0$, are identical with the corresponding partial derivatives (η) is fixed) to the left of the characteristic $\xi = 0$.

Now to prove the assertion made above, we must find $_\xi \equiv (\partial \tau / \partial \xi)_\eta$ as a function of η with $\xi = 0$ and $\eta < 0$, i.e. above the point o. Here τ denotes, for any c^--characteristic, the distance measured along it (also down the shock) from the characteristic $\xi = 0$. Taking into account the definition of ξ and the fact that the shock touches the c^--characteristic at the point o, we find that $\tau_{\xi 0} = 1$.

To find $\tau_\xi\,(0, \eta)$, we shall write the equations of the characteristics in the form

$$x_\eta = y_\eta \operatorname{ctg}(\vartheta - \alpha), \quad x_\xi = y_\xi \operatorname{ctg}(\vartheta - \alpha).$$

We differentiate the first equation with respect to ξ and the second with respect to η, we subtract one from the other, and use the equation $y_\xi = \tau_\xi \sin(\vartheta - \alpha)$ and the results of its differentiation with respect to η to eliminate y_ξ and $y_{\xi\eta}$. Let us find the derivatives of ϑ and α with respect to η and ξ, taking into account the fact that $\eta = I^-$, the formulas (2.1) and the expression

$$\alpha_p \equiv \left(\frac{\partial \alpha}{\partial p}\right)_{s,\,H} = \frac{\operatorname{ctg}\alpha}{\rho q^2}\left(\frac{\varphi}{2\cos^2\alpha} - 1\right), \quad \varphi = \varphi\,(p, s) = \rho^3 a^4 \omega_{pp} \tag{2.6}$$

which follows from the definition of the Mach angle, total enthalpy H and speed of sound, and the known thermodynamic relationships. In (2.6) $\omega = 1/\rho$ is the specific volume and $\omega_{pp} = (\partial^2\omega / \partial p^2)_s$. For a real gas we have $\varphi = \varkappa + 1$ where $\varkappa$ is the adiabatic index. Finally we find that the variation of τ_ξ on the characteristic $\xi = 0$ is described by the equation

$$\tau_{\xi\eta} - \frac{\varphi \operatorname{ctg} 2\alpha}{4\cos^2\alpha}\,\tau_\xi = y_\eta Q, \quad Q = \frac{-\varphi I_\xi^+}{4\cos^2\alpha \sin 2\alpha \sin(\vartheta + \alpha)} \tag{2.7}$$

where the coefficients of τ_ξ and $y_\eta Q$ are computed at $\xi = 0$ and are therefore known functions of η. According to (2.5), I_ξ^+ is defined when $\xi = 0$, by the first term. In the case in question the term is negative since the c^+-characteristics form a rarefaction wave /1, 2/. Integrating (2.7) and taking the initial condition $\tau_\xi = \tau_{\xi 0} = 1$ into account we obtain, for $\eta = 0$,

$$\tau_\xi = E\,(\alpha, \alpha_0)\left\{1 + \int_0^y Q\,(y')\,E_1\,(\alpha', \alpha_0)\,dy'\right\} \tag{2.8}$$

$$E\,(\alpha, \alpha_0) = E_1^{-1}\,(\alpha, \alpha_0) = \exp\left\{-\int_{\alpha_0}^{\alpha} P\,(\alpha')\,d\alpha'\right\}, \quad P\,(\alpha) = \frac{\varphi \operatorname{ctg} 2\alpha}{\varphi - 2\cos^2\alpha}\,.$$

In the case of a real gas for which, as has already been shown, $\varphi = \varkappa + 1$,

$$E(\alpha, \alpha_0) = \left(\frac{\sin\alpha_0}{\sin\alpha}\right)^{\varkappa_1}\left(\frac{\cos\alpha_0}{\cos\alpha}\right)^{1/2}\left(\frac{\varkappa - 1 + 2\sin^2\alpha}{\varkappa - 1 + 2\sin^2\alpha_0}\right)^{\varkappa_2}, \quad \varkappa_1 = \frac{\varkappa+1}{2(\varkappa-1)}, \quad \varkappa_2 = \frac{\varkappa}{2(\varkappa-1)}.$$

Since $I_\xi^+ < 0$ when $\xi = 0$, φ is positive for the "usual" gases and the quantity $\sin(\vartheta + \alpha)$ on the characteristic $\xi = 0$ above the point o is also positive, it follows that $Q > 0$ and according to (2.8), τ_ξ remains positive when $\alpha_0 \leqslant \alpha < \pi/2$ during the motion towards the SL. Therefore the derivatives of I_τ^+ and $I_{\tau\tau}^+$ behave in the same manner as I_ξ^+ and $I_{\xi\xi}^+$ for which the estimates (2.5) hold.

From amongst the coefficients (2.4), B is determined in the simplest manner. Namely, remembering that according to the last equation of (2.4), $[I^+] \sim [p]^3$ we obtain from (2.1), by virtue of the definition $\eta \equiv I^-$, we have at once $B = -(\rho q^2 \operatorname{tg}\alpha)_0 / 2$. To find the coefficient C we integrate the equation of the c^--characteristic $dx / dy = \operatorname{ctg}(\vartheta - \alpha)$ respectively from a and from b, to c. We take into account the "narrowness" of the triangle abc, the small variation in I^+ on the segments ac and bc where $\eta = \eta_a$ and $\eta = \eta_b = -\eta_a$ and the fact that the coordinates of the points a and b are found from (2.3) with $\eta_b = -\eta_a$. Equating the values of x_c found in this manner for both characteristics, we obtain an expression for y_c which yields, apart from higher-order infinitesimals, y_c as a linear function of $\eta_a^2 = [\eta]^2 / 4$. Next we use y_c to find, with the same accuracy, from any equation for x_c, the magnitude of the latter (and $\sim [\eta]^2$). Having found $\tau_c = \sqrt{x_c^2 + y_c^2}$ and neglecting the index c, we arrive at the second formula of (2.4) with the coefficient $C = -\{-\varphi / (A \sin 2\alpha \cos\alpha)\}_0^{1/2}$.

Here φ is the same as in (2.6). In the cases of an "ordinary" gas, which concerns us here, $\varphi > 0$ and when $A < 0$, which occurs as we already noted in the present situation, the expression within the braces is positive. The minus sign preceding it is taken because in (2.4) $[\eta] < 0$ and $\tau^{1/2}$ as well as τ are positive.

To find the third coefficient of (2.4) we use (2.6) and its corollary

$$\alpha_{pp} \equiv \left(\frac{\partial^2\alpha}{\partial p^2}\right)_{s,\,H} = \frac{\operatorname{ctg}\alpha}{\rho^2 q^4}\left(\frac{\varphi}{\cos^2\alpha} - 2\right)\left(1 - \varphi\,\frac{\operatorname{ctg}2\alpha}{\sin 2\alpha}\right) + \frac{\varphi_p}{\rho q^2 \sin 2\alpha} \tag{2.9}$$

in which $\varphi_p = (\partial\varphi/\partial p)_s$. For a real gas we have $\varphi_p = 0$.

Taking into account (2.6) and (2.9) and the fact that $(p_\pm - p_0) = \pm[p] / 2$, a $[s] = O([p]^3)$, we obtain from the expression for I^+ (the integral is taken for $s = s_0$)

$$[I^+] = [\vartheta] + \int_{p_-}^{p_+} \frac{\operatorname{ctg}\alpha}{\rho q^2}\, dp + o([p]^3) = [\vartheta] + \left(\frac{\operatorname{ctg}\alpha}{\rho q^2}\right)_- [p] + \tag{2.10}$$

$$\frac{1}{2}\left(\frac{\operatorname{ctg}\alpha}{\rho q^2}\right)_{p-}[p]^2 + \frac{1}{6}\left(\frac{\operatorname{ctg}\alpha}{\rho q^2}\right)_{pp}[p]^3 + o([p]^3),$$

$$\left(\frac{\operatorname{ctg}\alpha}{\rho q^2}\right)_p = \frac{2\operatorname{ctg}\alpha}{\rho^2 q^4}\left(1 - \frac{\varphi}{\sin^2 2\alpha}\right), \quad \left(\frac{\operatorname{ctg}\alpha}{\rho q^2}\right)_{pp} =$$

$$\frac{\operatorname{ctg}\alpha}{\rho^3 q^6}\left\{4 - \frac{1}{\sin^2\alpha} - \frac{\varphi}{\sin^2 2\alpha}\left(4 - \frac{\cos 2\alpha}{\sin^2\alpha}\right) + \right.$$

$$\left. 2\varphi^2\,\frac{1 + 2\cos 2\alpha}{\sin^4 2\alpha}\right\} - \frac{\varphi_p \operatorname{ctg}\alpha}{\rho^2 q^4 \sin^2 2\alpha}.$$

Further, from the relations on the shock we obtain, one after the other, $[s]$ and $\operatorname{ctg}(\vartheta_- - \sigma)$ where σ is its angle of inclination to the x axis, and then we find $[\vartheta]$ as a function of $[p]$, using the following relations:

$$a^2 = \frac{\rho h_\rho}{1 - \rho h_p}, \quad \omega_s = \frac{-T}{\rho^2 h_\rho}, \quad \omega_{ppp} \equiv \left(\frac{\partial^3\omega}{\partial p^3}\right)_s = \frac{\rho a^2 \varphi_p - (1 - 2\varphi)\varphi}{\rho^4 a^8}$$

in which $h = h(p, \rho)$ is the specific enthalpy, $\omega_s = (\partial\omega / \partial s)_p$, T is the absolute temperature, $h_p = (\partial h / \partial p)_\rho$ and $h_\rho = (\partial h / \partial\rho)_p$. The relations in question are

$$[s] = \left(\frac{\omega_{pp}}{12T}\right)_- [p]^3 + o([p]^3), \quad \operatorname{ctg}(\vartheta_- - \sigma) = \left\{1 - \frac{\varphi\,[p]}{\rho q^2 \sin^2 2\alpha} - \right.$$

$$\left.\left(\chi + \frac{\varphi}{4\cos^2\alpha}\right)\frac{[p]^2}{2\rho^2 q^4 \sin^3\alpha \sin^2 2\alpha} + o([p]^2)\right\}_- \operatorname{ctg}\alpha_-,$$

$$[\vartheta] = \left\{-1 + \left(\frac{\varphi}{\sin^2 2\alpha} - 1\right)\frac{[p]}{\rho q^2} + \left(\frac{\operatorname{ctg}^2\alpha}{3} - 1 + \right.\right.$$

$$\left.\left.\frac{\varphi}{\sin^2 2\alpha}\left(1 - \frac{1}{2\sin^2 2\alpha} + \frac{\beta - 4\varphi}{6\sin^2\alpha}\right)\right)\frac{[p]^2}{\rho^2 q^4}\right\}_- \left(\frac{\operatorname{ctg}\alpha}{\rho q^2}\right)_- [p] + o([p]^3)$$

$$\chi = \{(\beta - 4\varphi)\varphi + \rho a^2 \varphi_p\} / 3, \quad \beta = (2\rho h_p - 1)(\rho h_p - 1)^{-1}$$

and for a real gas we have $\beta = \varkappa + 1$. The first of the above relations is well-known. The second and third relations lead, in particular, to a result which is also known and has already been mentioned, here, namely the inclination of a weak shock wave. finally, substituting the expression for $[\vartheta]$ into (2.10) we obtain the formula for the coefficient D from (2.4)

$$D=\left\{\frac{\operatorname{ctg}^2\alpha}{6}-\frac{1}{2}+\frac{\varphi}{\sin^2 2\alpha}\left(\frac{2}{3}+\frac{1}{2\sin^2 2\alpha}+\frac{\mu}{6\sin^2\alpha}-\right.\right.$$
$$\left.\left.\varphi\,\frac{1+4\cos^2\alpha}{3\sin^2 2\alpha}\right)\right\}_0\left(\frac{\operatorname{ctg}\alpha}{\rho^3 q^6}\right)_0,\quad \mu=\frac{\rho h_p}{\rho h_p-1}+\frac{\rho a^2\varphi_p}{\varphi}$$

According to the expressions for B and C obtained earlier, the product BC is positive in the case discussed here. Therefore, the sign of the coefficient G in (2.5) is the same as the sign of D. Computations carried out for a real gas when $\varphi=\varkappa+1$, $\varphi_p=0$ and $\mu=\varkappa$ with $1\leqslant\varkappa\leqslant 5/3$ have shown that $(0\leqslant\alpha_0\leqslant\pi/2)\,D$ over the whole possible range of Mach angles and hence G is less than zero. Therefore, when $\xi>0$, the last terms on the right-hand sides of (2.5) are also negative and tend in the second equation to $-\infty$ as $\xi\to+0$. This corresponds to an additional rarefaction wave propagating from the point o towards the sonic line. This makes possible the reflection of the second "foot" of the shock closing the LSZ from the SL. The resulting flow pattern is shown in Fig.3. The figure depicts, apart from the c^+-characteristic passing through the point o, the shocks and the "basic" sonic line, with an "additional" supersonic zone containing a bundle of c^+-characteristics. Such a zone appears to the right of the point of interaction between both feet of the shock, since the shock polars do not intersect on the transonic line /9/.

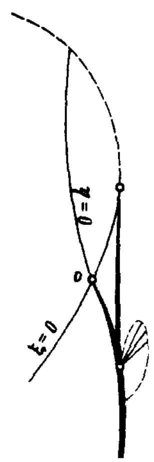

Fig.3

Thus we see that irrespective of the smoothness of the streamlined body, a configuration of shocks is possible in which the "right foot" of the closing shock originates at the sonic line although it is caused by the intersection of the c^- characteristics and an appearance of the "left foot" of the shock within the supersonic zone. We shall call a shock of this type the "inverse" λ-type shock. The configuration discussed above, on one hand, makes it necessary to bring in the results concerning the reflection of the singularities from the sonic line, which seemed to be relevant only to the non-smooth contours, and, on the other hand, it provides a fresh insight into the efforts of Frankl and his followers who sought a natural solution with a shock orginating at the sonic line for smooth bodies.

The author thanks G.G. Chernyi, O.S. Ryzhov, V.N. Diesperov, Yu.B Lifshitz and I.A. Chernov for valuable comments, and V.L. Grigorenko for computing the coefficient D numerically.

REFERENCES

1. NIKOL'SKII A.A. and TAGANOV G.I., Motion of a gas in a local supersonic zone and certain conditions of disintegration of a potential flow. PMM Vol.10, No.4, 1946.
2. NIKOL'SKII A.A., Theoretical Studies in Fluid and Gas Mechanics. Tr. TsAGI, Vol.2122, 1981.
3. FRANKL' F.I., On the formation of shock waves in subsonic flows with local supersonic velocities. PMM Vol.11, No.1, 1947.
4. FRANK' F.I., Collected Works in Gas Dynamics. Moscow, Nauka, 1973.
5. BUSEMANN A., The drag problem at high supersonic speeds. J. Aeronaut. Sci., Vol.16, No.6, 1949.
6. GUDERLEY K., The necessity for the existence of shock waves in mixed flows. In book: Problems of Mechanics. Moscow, Izd-vo inostr. lit. 1955.
7. MORAWETZ C.S., On the non-existence of continuous transonic flows past profiles. Communs. Pure and Appl. Math. I, Vol.9, No.1, p.45-68, 1956; II. Vol.10, No.1, p.107-131; 1957; III. Vol.11, No.1, 129-144, 1958.
8. LANDAU L.D. and LIFSHITZ E.M., Flow singularities studied using the Euler-Tricomi equations. Dokl. Akad. Nauk SSSR, Vol.96, No.4, 1954.
9. GUDERLEY K., The Theory of Transonic Flows. Pergamon Press, 1962.
10. FRANKL' F.I., Example of a transonic flow with a region of supersonic velocities bounded from downstream by a shock wave terminating within the flow. PMM Vol.19, No.4, 1955.
11. FRANKL' F.I., Flow past a profile of a subsonic stream witha supersonic zone bounded by a recilinear shock wave. PMM Vol.20, No.2, 1956.
12. FRANKL' F.I., Flow past a profile of a subsonic stream with a local supersonic zone bounded by a curved shock wave. PMM Vol.21, No.1, 1957.
13. BIIBOSUNOV I., An example of a transonic flow of gas with a region of supersonic velocities bounded downstream by a curved shock wave terminating within the flow. PMM Vol.22, No.3, 1958.
14. FRANKL' F.L., On the problem of symmetrical flow past a given symmetric profile in the case of subsonic velocity at infinity and local supersonic velocities. PMM Vol.23, No.4, 1959.

15. FRANKL' F.I., Investigations in the region of transonic flows. Inzh. zh. Vol.1, No.1, 1961.
16. LANIN I.N., Approximate construction of a subsonic flow past a given profile, with a local supersonic region terminated by a shock wave. Izv. vuzov. Aviats. tekhnika, No.3, 1961.
17. BIIBOSUNOV I. and RYSKULOV A., Plane parallel transonic flow with a discontinuity. Izv. Akad. Nauk SSSR, MZhG, No.4, 1970.
18. CHERNOV I.A. and SHEVYREV S.P., On the first Frankl case of a flow in the local supersonic zone with a shock wave. In book: Aerodynamika, Ed.2, Saratov, Izd-vo Sarat. un-ya, 1973.
19. GERMAIN P. and GILLON G., Écoulements transsoniques an voisinage d'un point de rencontre d'une onde de choc et d'une ligne sonique. Publ. O.N.E.R.A., No.102, 1961.
20. GOR'KOV L.P. and PITAEVSKII L.P., Appearance of a shock wave when a weak discontinuity is reflected from the sonic line. Dokl. Akad. Nauk SSSR, Vol.144, No.2, 1962.
21. LIFSHITZ YU.B., and RYZHOV O.S., On certain exact solutions of equations of transonic gas flows. Zh. vychisl. matem. i matem. fiz., Vol.4, No.5, 1964.
22. GERMAIN P., Écoulements transsoniques homogénes. In: Progress in Aeronautical Sciences. Vol.5, Oxford, e.a.: Pergamon Press 1964.
23. LIFSHITZ YU.B., On the flow near the point of contact between the sonic line and shock wave. Inzh. zh. Vol.5, No.1, 1965.
24. TSVETKOV A.P. and CHERNOV I.A., Selfsimilar transonic flows analytic on the limit characteristics. In book: Aerodinamika, Ed.1(4). Saratov, Izd-vo Sarat. un-ta, 1972.
25. TSVETKOV A.P., Flow in a local supersonic zone with a shock wave. In book: Aerodinamika, Ed.2(5), Aratov, Izd-vo Sarat, un-ta, 1973.
26. TSVETKOV A.P. and CHERNOV I.A., The problem of a local supersonic zone in the class of selfsimilar flows. Uch. zap. TsAGI, Vol.5, No.2, 1974.
27. MURMAN E.M., Analysis of embedded shock waves calculated by relaxation methods. AIAA Journal, Vol.12, No.5, 1974.
28. MORETTI G., Accuracy and efficiency in the numerical analysis of transonic flows. In: Symposium Transonicum. 2. Göttingen, 1975. B.: Springer, 1976.
29. YU N.J. and SEEBASS A.R., Inviscid transonic flow computations with shock fitting. In: Symposium Transonicum. 2. Göttingen, 1975. B.: Springer, 1976.
30. YERE"MENKO V.A. and RYZHOV O.S., On the flow in a local supersonic zone on the profile of a wing of infinite span. Dokl. Akad. Nauk SSSR, Vol.240, No.3, 1978.
31. IVANOV V.A. and CHERNOV I.A., Frankl solutions generalized to the case of subsonic flow away from the profile. In book: Aerodinamika, Ed.7(10). Saratov, Izd-vo Sarat. un-ta, 1979.
32. IVANOV V.A. and CHERNOV I.A., On the problem of the appearance of a shock wave inside a local supersonic zone. PMM Vol.43, No.6, 1979.
33. IVANOV V.A. and CHERNOV I.A., Example of the solution of transonic equations for a shock-free flow past a symmetric profile. PMM Vol.46, No.1, 1982.
34. LANDAU L.D. and LIFSHITZ E.M., Mechanics of Continuous Media. Moscow, Gostekhizdat, 1953.
35. FRIEDRICHS K.O., Formation and decay of shock waves. Communs Appl. Math., Vol.1, No.3, 1948.
36. CHERNYI G.G., Appearance and form of the shock surfaces in gas flows. In book: Teoreticheskaya gidromekhanika, Ed.2, Moscow, Oborongiz, 1952.

Translated by L.K.